3

90638

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NEW ZEALAND QUALIFICATIONS AUTHORITY
MANA TOHU MĀTAURANGA O AOTEAROA

Level 3 Calculus, 2008

90638 Manipulate real and complex numbers, and solve equations

Credits: Five
9.30 am Tuesday 18 November 2008

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

Make sure you have a copy of the Formulae and Tables Booklet L3–CALCF.

You should answer ALL the questions in this booklet.

Show ALL working for ALL questions.

If you need more space for any answer, use the page(s) provided at the back of this booklet and clearly number the question.

Check that this booklet has pages 2–11 in the correct order and that none of these pages is blank.

YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.

For Assessor's use only	Achievement Criteria	
Achievement	**Achievement with Merit**	**Achievement with Excellence**
Manipulate real and complex numbers, and solve equations.	Solve more complicated equations.	Solve problem(s) involving real or complex numbers.
	Overall Level of Performance	

You are advised to spend 40 minutes answering the questions in this booklet.

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QUESTION ONE

Write $\dfrac{3+2\sqrt{3}}{-4+\sqrt{3}}$ in the form $a+b\sqrt{3}$ where a and b are real numbers.

QUESTION TWO

(a) Solve for x:

$$\log_2(x+3) = \log_2(x) + \log_2(3)$$

(b) Solve for x in terms of p:

$$2^{x-p} = 16^x$$

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QUESTION THREE

(a) Given complex numbers $u = 2 - i$ and $w = -3 + 4i$, find the following in form $a + bi$:

(i) u + 3w

(ii) $\dfrac{u}{w}$

(b) $v = \text{cis}\left(\dfrac{\pi}{5}\right)$ and $s = 2\text{cis}\left(\dfrac{3\pi}{10}\right)$

Find the product vs

QUESTION FOUR

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Solve the following equation for x in terms of k:

$$\ln(3x-2)-\ln(x+3)=2\ln k$$

QUESTION FIVE

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$z = 2 - 3\mathrm{i}$ is one of the roots of the polynomial equation $z^3 - pz^2 + qz - r = 0$, where p, q and r are real.

Find the real root in terms of p or r.

QUESTION SIX

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The average of any two real numbers, x and y, can be calculated in various ways.

The arithmetic mean, A, is found by halving the sum of the two numbers: $A = \dfrac{x+y}{2}$

The geometric mean, G, is the square root of the product of the two numbers: $G = \sqrt{xy}$

The harmonic mean, H, is twice the reciprocal of the sum of the reciprocals of the two numbers:

$$H = \frac{2}{\frac{1}{x} + \frac{1}{y}}$$

Prove that $\sqrt{\dfrac{A.H}{2}} - \sqrt{2}\,G + \dfrac{G}{\sqrt{2}} = 0$ for all x and y.

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Question Seven is on the following page.

QUESTION SEVEN

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(a) Use de Moivre's theorem to find all solutions for z in the equation $z^3 = w$, where w is any real number.

(b) If w is any real number, the equation $z^k = w$ has k roots, $k \in N$.

Show that the sum of the k roots is zero.

Hint: you may find useful the formula for the sum of a geometric sequence.

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Extra paper for continuation of answers if required.
Clearly number the question.

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